Sample Paper 7: Paper 1

Question 5 (25 marks)

Question 5 (a)

$f'(x) = y = ax^2 + bx + c$

$(0,\ 6) \in f'(x) \Rightarrow a(0)^2 + b(0) + c = 6$

$\therefore c = 6$

$(1,\ 0) \in f'(x) \Rightarrow a(1)^2 + b(1) + 6 = 0$

$\therefore a + b = -6....(\mathbf{1})$

$(3,\ 0) \in f'(x) \Rightarrow a(3)^2 + b(3) + 6 = 0$

$\therefore 9a + 3b = -6$

$3a + b = -2....(\mathbf{2})$

$(\mathbf{2}) - (\mathbf{1}) : 2a = 4 \Rightarrow a = 2$

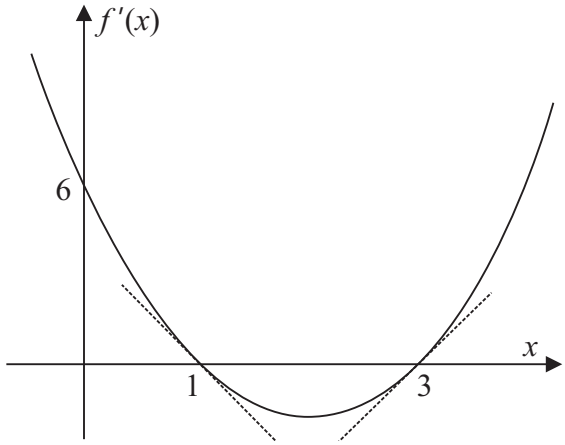
$(2) + b = -6....(\mathbf{1}) \Rightarrow b = -8$

$\therefore f'(x) = y = 2x^2 - 8x + 6$

Question 5 (b)

Local minimum at $x = 3$ as $\dfrac{dy}{dx} = 0$ and the slope is positive.

Local maximum at $x = 1$ as $\dfrac{dy}{dx} = 0$ and the slope is negative.



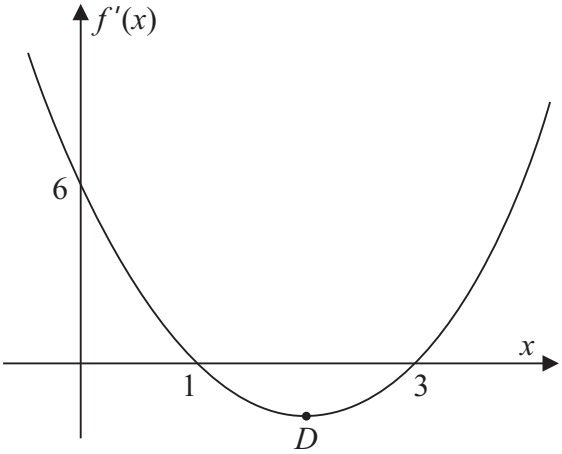
Question 5 (c)

$f'(x) = y = 2x^2 - 8x + 6$

$f''(x) = 4x - 8$

$f''(x) = 0 \Rightarrow 4x - 8 = 0$

$\therefore x = 2$



Question 5 (d)

$\dfrac{dy}{dx} = 2x^2 - 8x + 6$

$y = \frac{2}{3}x^3 - 4x^2 + 6x + c$

$x = 0,\ y = 0 \Rightarrow c = 0$

$\therefore y = f(x) = \frac{2}{3}x^3 - 4x^2 + 6x$

Question 5 (e)

$f(x) = \frac{2}{3}x^3 - 4x^2 + 6x$

$f(3) = \frac{2}{3}(3)^3 - 4(3)^2 + 6(3) = 18 - 36 + 18 = 0$

Local minimum: (3, 0)

$f(1) = \frac{2}{3}(1)^3 - 4(1)^2 + 6(1) = \frac{2}{3} - 4 + 6 = \frac{8}{3}$

Local maximum: $(1,\ \frac{8}{3})$

$f(2) = \frac{2}{3}(2)^3 - 4(2)^2 + 6(2) = \frac{16}{3} - 16 + 12 = \frac{4}{3}$

Point of inflection: $(2,\ \frac{4}{3})$